spring up. It is the more remarkable that he held such a view, when we consider that early in his career he made a notable contribution to space analysis. Starting from Rodrigues' formulae for the rotation of a solid body, he arrived at the quaternion formula, and was anticipated by Hamilton only by a few months. But Cayley took a Cartesian view of analysis to the last, as is evident from the chapter which he contributed to Tait's *Treatise on Quaternions*. His aim there is to give an analytical theory of quaternions. Hamilton's aim on the other hand was to give a quaternionic theory of analysis. The difference is brought out still more strikingly in a paper printed in the last number of the *Proceedings of the Royal Society of Edinburgh*.

In 1889 the Cambridge University Press commenced the re-publication of his mathematical papers in a collected form. It was calculated that they would occupy 10 quarto volumes; 7 volumes have already appeared; and it is believed that 12 volumes will be required. No mathematician has ever had his works printed in a more handsome manner. In addition he is the author of a separate work on *Elliptic Functions*.

Space fails to enumerate the honors which he received from Universities and Scientific Academies both of the Old and of the New World. But we may mention specally, that from the Royal Society he received a Royal Medal and a Copley Medal; from the Mathematical Society of London the first De-Morgan Medal; and at the instance of the President and Members of the French Academy he was made an Officer of the Legion of Honour.

On the 26th of January he died at Cambridge. His body was laid to rest in Mill Road Cemetery in the presence of official representatives from foreign countries and many of the most illustrious philosophers of England. His spirit still speaks to us from his works, and will continue to speak to many succeeding generations.

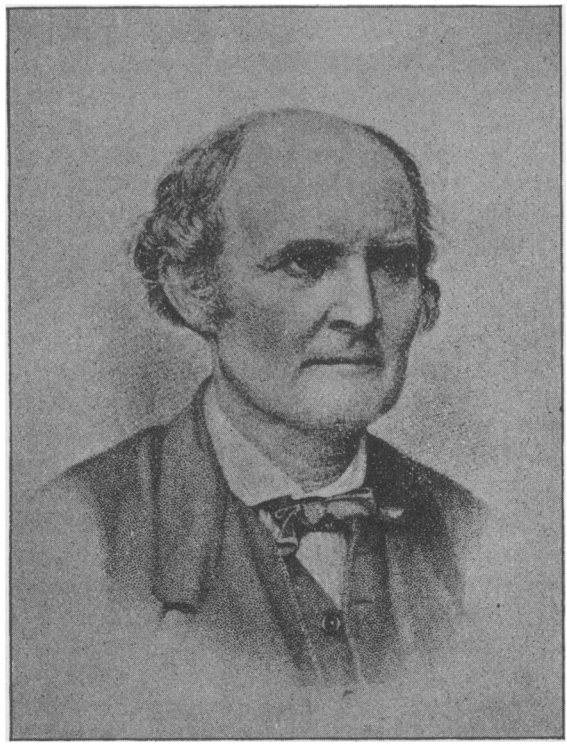
II. BIOGRAPHY.

ARTHUR CAYLEY.

BY DR. GEORGE BRUCE HALSTED.

On January 26th, 1895, after long suffering, passed away Professor Cayley, one of the very greatest masters ever known to the world of pure mathematics.

Of the great quaternion of Senior Wranglers of 1840 Leslie Ellis, 1841 Stokes, 1842 Cayley, 1843 Couch Adams, the second alone now remains.



ARTHUR CAYLEY.

Arthur Cayley was born on August 16th, 1821. His ancestor Roger de Cailli was a great lord in the reign of Henry II. His grandfather and father were both merchants in St. Petersburg. His father married a Russian, and though born in England, Cayley's mother tongue was Russian, and his features had a Russian cast. Like so many Russians, he spoke most European languages well.

Can there be anything in what has so often been cited as fact, that in the Russian race alone the brain of the woman equals that of the man in size and weight?

Arthur Cayley was a pupil of King's College School, London, and entered Trinity College, Cambridge, already a well-equipped mathematician, at the age of 17. In 1842 he took the two highest honors in the University of Cambridge, he was Senior Wrangler and First Smith's Prizeman. At that time, more than half a century ago, the Senior Wrangler was almost always as a matter of course a Johnian, so a Trinity Senior Wrangler was apt to be an object of curiosity. One of his college mates describes him at that date as a crooked little man, in no respect a beauty, and not in the least a beau. On the day of his triumph, when he was to receive his hard-earned honors in the Senate House, some of his friends combined their energies to dress him, and put him to rights properly, so that his appearance might not be altogether unworthy of his exploits and his College. He was already a man of much varied information, and that on some subjects the very opposite of scientific; for instance he was well up in all the current novels, an uncommon thing at Cambridge, where novel-reading then was not one of the popular weaknesses. His Johnian competitor for first place was a fearfully hard student, and had once worked *twenty hours a day* for a week together at a College examination. But now he almost broke down from over exertion just as the time of trial was coming on, and actually carried a supply of ether and other stimulants into the examination, in case of accidents. Nevertheless he made a good fight of it, and having great *pace* as well as *style* in addition to his knowledge, beat Cayley a little on the bookwork, but was beaten two hundred marks in problems, which decided the contest.

One of the low bookwork papers to which three hours were allotted happening to be rather shorter than usual, the man from St. John's, either as a bit of bravado to frighten his opponent, or because having done all that could be done he had no reason for waiting longer, came out at the expiration of two hours, having floored the paper in that time. His early exit did not escape notice, and the same evening a Trinity Senior Soph rushed up in great fear to the room of his friend, on whom the hopes of the College depended. "Cayley! Cayley! they tell me S— floored the paper this afternoon in two hours. Is it so?" The mathematician, who was refreshing himself after the fatigues of the day with the innocent and economical luxury of a footbath, looked up at the querist from his tub with the equanimity of a Diogenes, and replied: "Likely enough he did. I floored it myself in two hours and a half". The examination for the Smith's Prizes which took place immediatly after the

result of the Mathematical Tripos was declared, had a similar result: Cayley beat his opponent, but with nothing to spare. The matter was very different the next year, when Couch Adams, the discoverer of Neptune, won not only easily, but had three thousand marks to the Second Wrangler's fourteen hundred, so that there was more numerical difference between them than between the Second Wrangler and the *spoon*, or last man. But this was produced by a singular case of fright or stampede which occurred at this examination. The man who would have been second, like Adams a Johnian, took fright when four of the six days were over, and actually ran away, not only from the examination but out of Cambridge, and was not discovered by his friends or family till some time after. Even as it was and without the last two days, he came out ninth in the list of wranglers. But even if Cayley had been beaten for first place, he might still have been equally as eminent as now: for has not Cambridge that other tremendous tetrad, Sylvester, Wm. Thomson, Clerk-Maxwell, Clifford, all Second Wranglers!

In 1841 Cayley published his first paper, thus commencing the astounding series of over 800 memoirs with which he so enriched his science. The collected edition of his works now being published by the University Press will extend to ten or more quarto volumes, a scientific monument equally unique in amount, range, and quality.

After his election to a Fellowship, which, as he was unwilling to take Holy Orders, could be only temporary, he studied conveyancing in London, and at Lincoln's Inn first met his greatest and lifelong friend and fellow genius Sylvester, for they had never met at Cambridge, where Sylvester was Second Wrangler in 1837.

He practised as a conveyancer for 14 years, but during this time his real occupation was pure mathematics, and in those years some of his most notable discoveries were made. The law was always drudgery to him. The superabundant verbiage of legal forms was always distasteful to him. He once remarked that "the object of law was to say a thing in the greatest number of words, of mathematics to say it in the fewest."

Cayley was a very gentle, sweet character. Sylvester told me that he never saw him angry but once, and that was when a messenger broke in on one of their interviews with a mass of legal documents, new business for Cayley. In an access of disgust, Cayley dashed the documents upon the floor.

In 1863 Lady Sadler's various trusts were consolidated, and a new Sadlerian Professorship of Pure Mathematics was created in the University of Cambridge, especially for Cayley. As chairman of the Association for Promoting the Higher Education of Women he did most to raise Newnham College to its present influential position.

In Cambridge he was accustomed to give the small classes of advanced students who were prepared to follow him no mere routine course, but, like the best German professors since Jacobi, the latest and highest work on which he was at the time engaged.

As early as 1852 he was a fellow of the Royal Society. In 1858 he

joined Sylvester and Stokes in starting the Quarterly Journal of Pure and Applied Mathematics. In 1882 he delivered a special course of lectures at the Johns Hopkins University, where Sylvester was still professor. Baltimore was then the apex not only of the Western Continent, but of the world, for Salmon soon after said that if European mathematicians had to elect themselves a head, it would be Cayley. In 1863 he married and settled permanently in Cambridge.

Cayley was assuredly the most learned and erudite of mathematicians. Of him it might be said, he knew everything, and he was the very last man who ever will know everything. I have heard Sylvester say that when he wished to know anything he simply asked Cayley, for to Sylvester it was not only often irksome to study what had been done by others, but impossible, since the very beginning of such study was sure to start in him a train of original thought and research which absorbed him irresistably. This wideness of knowledge made Cayley invaluable as a mathematical referee. To the Royal Society, the Mathematical Society, the Royal Astronomical Society, the Cambridge Philosophical Society he was long the principal adviser as to the merits of mathematical papers presented for publication. Cayley's erudition gave his originality always the most fertile fields.

In 1841 the wonderful George Boole, the creator of algorithmic logic, made use of a simple case of what we would now call *invariance* in linear substitutions. Then Cayley set himself the problem to determine *a priori* what functions of the coefficients of a given equation possess this property. He called such functions hyperdeterminants, until Sylvester the mathematical Adam, who names the creatures, called them *invariants*. Substitutions and invariance are now the heart of the very latest analytic mathematics, and have received an extraordinary transformation and development at the hands of Sophus Lie.

Again, the idea that any metrical property in geometry could be looked upon as a projective relation in a particular configuration began to occur in the French school. For example Laguerre in 1853 so expresses an angle. But in 1859 in his sixth memoir on Quantics Cayley published his solution of the general problem he had set himself of finding a general theory of projective metries of which ordinary metrics should be a special case; thus breaking down the distinction between pure positional or descriptive geometry and the ordinary metrical geometry by merging all into projective geometry.

Remembering that von Staudt had founded cross-ratio on a pure projective basis in his theory of the *Wurf*, entirely without using measurement in the ordinary sense (direct comparison as to size by congruence), Klein saw that Cayley's theory of projective measurement leads directly to the three possible cases of geometry, Euclidean and non-Euclidean, which he called parabolic, elliptic, hyperbolic. The hyperbolic is the now well-known non-Euclidean geometry of Lobachevsky and Bolyai. Thus Cayley's doctrine of "the absolute", already greatly admired, was given additional importance, and its creation will ever rank as one of the very greatest of his achievements.

As a third epoch-making production of his fertile and tireless genius we may mention the theory of matrices, on which multiple algebra is based. In this, as in the theory of invariants, Sylvester was his most brilliant co-adjutor. Cayley was a devoted admirer of Euclid. In his great address as President of the British Association, speaking of Greek mathematics he says: "But the earliest extant writings are those of Euclid (B. C. 285).

There is hardly anything in mathematics more beautiful than his wondrous fifth book; and he has also in the seventh, eighth, ninth, and tenth books fully and ably developed the first principles of the theory of numbers, including the theory of incommensurables". In the same address he says: "It is well known that Euclid's twelfth axiom, even in Playfair's form of it, has been considered as needing demonstration; and that Lobachevsky constructed a perfectly consistent theory, wherein this axiom was assumed not to hold good, or say a system of non-Euclidean plane geometry. There is a like system of non-Euclidean solid geometry. Riemann's view was that having *in intellectu* a more general notion of space (in fact a notion of non-Euclidean space), we learn by experience that space (the physical space of our experience), is, if not exactly, at least approximately, Euclidean space. But suppose the physical space of our experience to be thus only approximately Euclidean space, what is the consequence which follows? Not that the propositions of geometry are only approximately true, but that they remain absolutely true in regard to that Euclidean space which has been so long regarded as being the physical space of our experience.

The three geometries (spherical, Euclidean, and Lobachevsky's) should be regarded as members of a system—viz., they are the geometries of a plane (two-dimensional) space of constant positive curvature, zero curvature, and constant negative curvature respectively; or again they are the plane geometries corresponding to three different notions of distance; in this point of view they are Klein's elliptic, parabolic, and hyperbolic geometries respectively."

But here this imperfect sketch must stop. Enough that his life furthered in the highest degree the aim of his university, in the words of his mother's compatriot Lobachevsky, "not only to enlighten the spirit with knowledge, but also to inculcate virtues, to implant a desire for glory, a feeling of nobility, justice, and honor, of strict and sacred honesty, that would resist all cases of temptation, apart from any fear of punishment."